## Line Shapes of the Z(4430)

Eric Braaten and Meng Lu

Physics Department, Ohio State University, Columbus, Ohio 43210, USA

(Dated: February 2, 2008)

The Belle Collaboration recently discovered the first manifestly exotic meson:  $Z^+(4430)$ , which decays into  $\psi'\pi^+$  and therefore has quark content  $c\bar{c}u\bar{d}$ . The proximity of its mass to the  $D_1\bar{D}^*$  threshold has motivated the interpretation of the  $Z^+$  as a charm meson molecule whose constituents are an S-wave superposition of  $D_1^+\bar{D}^{*0}$  and  $D^{*+}\bar{D}_1^0$ . If this interpretation is correct, the small ratio of the binding energy of the  $Z^+$  to the width  $\Gamma_1$  of its constituent  $D_1$  can be exploited to predict properties of its line shapes. Its full width at half maximum in the channel  $\psi'\pi^+$  should be approximately  $\sqrt{3}\Gamma_1\approx 35$  MeV, which is consistent with the measured width of the  $Z^+$ . The  $Z^+$  should also decay into  $D^*\bar{D}^*\pi$  through decay of its constituent  $D_1$ . The peak in the line shape for  $D^*\bar{D}^*\pi$  should be at a higher energy than the peak in the line shape for  $\psi'\pi^+$  by about  $\Gamma_1/\sqrt{12}\approx 6$  MeV. The line shape in  $D^*\bar{D}^*\pi$  should also be broader and asymmetric, with a shoulder on the high energy side that can be attributed to a threshold enhancement in the production of  $D_1\bar{D}^*$ .

PACS numbers: 12.38.-t, 12.39.St, 13.20.Gd, 14.40.Gx

Hadrons are bound states of quarks and antiquarks that feel the strong force of quantum chromodynamics. The ordinary hadrons are baryons which contain 3 quarks, antibaryons which contain 3 antiquarks, and ordinary mesons which contain a quark and an antiquark. Any other type of hadron is called exotic. Hundreds of hadrons have been discovered, but none were manifestly exotic until the recent discovery of the  $Z^{\pm}(4430)$  by the Belle Collaboration [1]. The  $Z^{+}$  decays into  $\psi'\pi^{+}$ , where  $\psi'$  is a charmonium meson whose constituents are a charm quark and antiquark  $(c\bar{c})$  and  $\pi^{+}$  is a charged pion whose constituents are a light quark and antiquark  $(u\bar{d})$ . Thus the  $Z^{+}$  is a manifestly exotic meson with quark content  $c\bar{c}u\bar{d}$ .

Since the  $Z^+$  contains two quarks and two antiquarks, it can be called a tetraquark meson. However its structure is still an open question. The proposed interpretations of the  $Z^+$  can be classified according to the clusterings of its constituents that appear in the dominant component of the wave function. If the  $Z^+$  is a charm meson molecule [2, 3, 4, 5, 6, 7], the dominant clusters are charm mesons with quark contents  $c\bar{u}$  and  $\bar{c}d$ . If the  $Z^+$  is a tetraquark composed of a diquark and an antidiquark [8, 9], the dominant clusters are cu and  $\bar{c}d$ . If the  $Z^+$  is a baryonium [10], the dominant clusters are cuq and  $\bar{c}d\bar{q}$ , where q is another light quark.

The mass of the  $Z^+$  is extremely close to the  $D_1\bar{D}^*$  threshold, where  $D_1$  and  $D^*$  are charm mesons with spin-parity quantum numbers  $J^P=1^+$  and  $1^-$ . This has motivated the proposal that  $Z^+$  is a charm meson molecule whose constituents are a superposition of the charm meson pairs  $D_1^+\bar{D}^{*0}$  and  $D^{*+}\bar{D}_1^0$  [2, 3, 4, 5, 6, 7]. Rosner argued that the quantum numbers  $J^P=0^-$  are favored because of the small energy release in the decay  $B^+\to K^0Z^+$  [2]. Meng and Chao used a model of charm meson rescattering to argue that decays of  $Z^\pm$  into

 $D^*\bar{D}^*\pi$  should dominate over charmonium decay modes such as  $\psi'\pi^{\pm}$  [3]. Their model can accomodate a substantial suppression of  $J/\psi \pi^{\pm}$  relative to  $\psi'\pi^{\pm}$  if  $J^P=1^-$ . Bugg suggested that the  $Z^{\pm}$  should produce a cusp at the  $D_1\bar{D}^*$  threshold in several other charm meson decay channels [4]. Liu, Liu, Deng, and Zhu used a variational approximation to argue that the one-pion-exchange potential between  $D_1$  and  $\bar{D}^*$  is not strong enough to bind them into  $Z^{\pm}$  [5]. Their results are not conclusive because their variational wavefunctions did not allow a significant contribution to the energy from the short-distance region where the potential is dominated by a  $1/r^3$  term.

If  $Z^+$  is a  $D_1\bar{D}^*$  molecule, the simplest possibility for its orbital angular momentum state is S-wave, in which case its  $J^P$  quantum numbers must be  $0^-$ ,  $1^-$ , or  $2^-$ . The tiny binding energy of the  $Z^+$  implies a strong resonant interaction between the charm mesons that produces an S-wave scattering length that is large compared to the range of their interaction. S-wave threshold resonances have universal properties that are determined by the large scattering length and are otherwise insensitive to shorter distance scales [11]. The width  $\Gamma_1$  of the  $D_1$  smears out the resonant interactions so that the universal features are not as distinctive. However  $\Gamma_1$  is small enough that we can exploit the universal features of the  $Z^+$  provided we take into account the width of its constituent.

The line shape of a resonance is the invariant mass distribution of its decay products. In this paper, we point out simple features of the line shapes of  $Z^{\pm}$  that follow from its identification as a weakly-bound S-wave charm meson molecule. Its line shape in  $D^*\bar{D}^*\pi$  will differ significantly from its line shape in  $\psi'\pi^{\pm}$  because the resonance overlaps with a threshold enhancement in the production of  $D_1\bar{D}^*$ . As a consequence, measurements of the

mass and width of the resonance in the  $D^*\bar{D}^*\pi$  channel will give larger values than in the  $\psi'\pi$  channel.

A similar phenomenon has been observed in the case of the X(3872), which was discovered by the Belle Collaboration in 2003 [12]. Its mass is extremely close to the  $D^{*0}\bar{D}^0$  threshold. This has motivated its interpretation as a charm meson molecule whose constituents are a superposition of  $D^{*0}\bar{D}^{0}$  and  $D^{0}\bar{D}^{*0}$ . In this case, it would be an exotic meson with quark content  $c\bar{c}u\bar{u}$ . Unlike the  $Z^{\pm}$ , it is not manifestly exotic because it can mix with  $c\bar{c}$  mesons through annihilation of the  $u\bar{u}$  pair. The X(3872) was discovered in the decay mode  $J/\psi \pi^+\pi^-$ . It can also decay into  $D^0\bar{D}^0\pi^0$  through the decay of its constituent  $D^{*0}$  or  $\bar{D}^{*0}$ . The measured mass of the resonance is higher in the  $D^0 \bar{D}^0 \pi^0$  channel than in the  $J/\psi \pi^+ \pi^$ channel by about 4 MeV [13, 14]. The width of the Xresonance also appears to be larger in the  $D^0\bar{D}^0\pi^0$  channel. The width in the  $D^0\bar{D}^0\pi^0$  channel was measured to be  $2.42\pm0.55$  MeV [13], but there is only an upper bound on the width in the  $J/\psi \pi^+\pi^-$  channel:  $\Gamma_X < 2.3 \text{ MeV}$ at 90% confidence level [12]. In Refs. [15, 16], the line shapes of X(3872) were analyzed under the assumption that it is a charm meson molecule. The observed mass difference was shown to be compatible with this interpretation. An alternative explanation of the mass difference between the two channels is that they correspond to two nearly degenerate tetraquark mesons [17].

We begin by summarizing the properties of the  $Z^+(4430)$ . It was discovered in the decay  $B^+ \to K^0 + Z^+$ . Its measured mass and width are [1]

$$M_Z = 4433 \pm 4 \pm 2 \text{ MeV},$$
 (1a)

$$\Gamma_Z = 45^{+18+30}_{-13-13} \text{ MeV}.$$
 (1b)

The mass in Eq. (1a) is close to the threshold for the charm mesons  $D^{*+}$  and  $\bar{D}_1^0$ , which have quark content  $c\bar{d}$  and  $\bar{c}u$  and spin-parity quantum numbers  $J^P=1^-$  and  $1^+$ . The difference between charm meson masses are measured more accurately than the masses themselves. If we combine the PDG values of the mass differences [18] with a recent precise measurement of the  $D^0$  mass by the CLEO Collaboration [19], we find that the  $D^{*+}\bar{D}_1^0$  threshold is at

$$M_{D_1^0} + M_{D^{*+}} = 4432.2 \pm 0.9 \text{ MeV}.$$
 (2)

This is not a sharp threshold because the  $D_1^0$  has a width  $\Gamma_1 = 20.4 \pm 1.7$  MeV [18]. The  $D_1^+ \bar{D}^{*0}$  threshold may differ by a few MeV, but the difference is negligible compared to  $\Gamma_1$ . The binding energy of the  $Z^+$  defined by the difference between the threshold in Eq. (2) and its measured mass in Eq. (1a) is

$$E_Z = -1 \pm 5 \text{ MeV}. \tag{3}$$

The central value is negative corresponding to a resonance or a virtual state, but the error bar is compatible with a bound state. The discovery decay mode is  $Z^+ \to \psi' \pi^+$ . The decay products  $\psi'$  and  $\pi^+$  have isospin

and G-parity quantum numbers  $I^G = 0^-$  and  $1^-$ , respectively. Thus the quantum numbers of the  $Z^+$  are  $I^G = 1^+$ . If the  $Z^+$  is a  $D_1\bar{D}^*$  molecule, the members of its isospin multiplet have the particle content

$$\begin{split} Z^+ &= \frac{1}{\sqrt{2}} \left( D^{*+} \bar{D}_1^0 - D_1^+ \bar{D}^{*0} \right), \\ Z^0 &= \frac{1}{2} \left( D_1^0 \bar{D}^{*0} - D^{*0} \bar{D}_1^0 + D_1^+ D^{*-} - D^{*+} D_1^- \right), \\ Z^- &= \frac{1}{\sqrt{2}} \left( D_1^0 D^{*-} - D^{*0} D_1^- \right). \end{split}$$

If the Z has  $J^P$  quantum numbers  $0^-$ ,  $1^-$ , or  $2^-$ , it has an S-wave coupling to  $D_1\bar{D}^*$ . The tiny binding energy in Eq. (3) then implies that it is an S-wave threshold resonance with universal properties. We proceed to describe the consequences for elastic scattering of its constituents. The amplitude  $\mathcal{A}$  for elastic scattering of a  $D_1\bar{D}^*$  pair in a resonant S-wave channel can be written as

$$\mathcal{A} = (2\pi/M_{1*})f(E) , \qquad (4)$$

where  $M_{1*}$  is the reduced mass for the  $D_1\bar{D}^*$  pair. The scattering amplitude f(E) depends only on the energy difference E between the invariant mass of the charm mesons and the  $D_1\bar{D}^*$  threshold. If there is a threshold S-wave resonance, the scattering amplitude for energy E near the threshold has a simple universal form:

$$f(E) = [-\gamma + \kappa(E)]^{-1} , \qquad (5)$$

where  $\kappa(E)=(-2M_{1*}E-i\epsilon)^{1/2}$  and  $\gamma$  is the inverse scattering length. The universal expression in Eq. (5) is valid when  $\kappa$  is large compared to  $\frac{1}{2}|r_s\kappa^2|$ , where  $r_s$  is the S-wave effective range. Since the long-range interactions of  $D_1$  and  $\bar{D}^*$  are dominated by pion exchange, we expect  $|r_s|\lesssim 1/m_\pi$ . Thus the universal region  $|E|<2/(M_{*1}r_s^2)$  should at least include the interval |E|<36 MeV. The simple expression for  $\kappa(E)$  with a branch point at E=0 applies if the constituents are stable. If the width  $\Gamma_1$  of  $D_1$  is taken into account, the branch point should be at  $E=-i\Gamma_1/2$ . Thus an expression for  $\kappa(E)$  that takes into account the width of the constituent  $D_1$  is [16]

$$\kappa(E) = \sqrt{-2M_{1*} \left(E + i\Gamma_1/2\right)}.\tag{6}$$

If E is real, a more explicit expression for  $\kappa(E)$  with the appropriate choice of branch cut is

$$\kappa(E) = \sqrt{M_{1*}} \left[ (E^2 + \Gamma_1^2/4)^{1/2} - E \right]^{1/2} - i\sqrt{M_{1*}} \left[ (E^2 + \Gamma_1^2/4)^{1/2} + E \right]^{1/2} . \quad (7)$$

The universal expression for the scattering amplitude given by Eqs. (5) and (7) can be motivated by unitarity. The imaginary part of f(E) can be written as

$$\operatorname{Im} f(E) = |f(E)|^{2} \operatorname{Im} \left[ \gamma - \kappa(E) \right] . \tag{8}$$

If  $\gamma$  is real and if  $\Gamma_1 = 0$ ,  $\text{Im}[\gamma - \kappa(E)]$  reduces to  $(2M_{1*}E)^{1/2}\theta(E)$ . In this case, Eq. (8) is simply the optical theorem that expresses the exact unitarity of elastic

scattering in the S-wave channel. The effects of inelastic scattering can be taken into account through the variables  $\operatorname{Im}\gamma$  and  $\Gamma_1$ . Since the dominant decay modes of the  $D_1$  are  $D^*\pi$  [20], the variable  $\Gamma_1$  takes into account inelastic scattering channels of the form  $D^*\bar{D}^*\pi$ . The variable  $\operatorname{Im}\gamma$  takes into account other inelastic scattering channels that do not involve the decay of a constituent of Z, such as  $\psi'\pi$ ,  $J/\psi\pi$ ,  $D^*\bar{D}$  and  $D^*\bar{D}^*$ . We refer to them as short-distance channels, because they proceed through intermediate states in which the  $D_1$  and  $\bar{D}^*$  have a separation that is small compared to  $1/|\gamma|$ . The imaginary part of  $\gamma$  can be partitioned into contributions from the individual short-distance channels:

$$Im\gamma = \sum_{C} \Gamma^{C}.$$
 (9)

To derive universal expressions for the line shapes of the  $Z^+$  in the decay  $B^+ \to K^0 + Z^+$ , we start from the optical theorem for the inclusive decay into  $K^0$  plus particles that couple to the  $Z^+$  resonance. If the difference E between the invariant mass of the resonating particles and the  $D_1\bar{D}^{*0}$  threshold is small enough, the invariant mass distribution can be written as

$$\frac{d\Gamma}{dE} \left[ B^+ \to K^0 + \text{resonant} \right] = 2\Gamma_B^K \text{ Im} f(E) , \qquad (10)$$

where  $\Gamma_B^K$  is a constant with dimension of energy that takes into account the probability for the constituents of  $Z^+$  to be created in the transition  $B^+ \to K^0$ . The energy distribution in Eq. (10) can be decomposed into the line shapes for individual resonant channels by inserting Eqs. (8) and (9). The line shape in the short-distance decay mode  $\psi'\pi^+$  is

$$\frac{d\Gamma}{dE} \left[ B^+ \to K^0 + \psi' \, \pi^+ \right] = 2\Gamma_B^K \left| f(E) \right|^2 \Gamma^{\psi'\pi} \, . \tag{11}$$

This line shape applies equally well to any other short-distance decay mode C, with the constant  $\Gamma^{\psi'\pi}$  replaced by  $\Gamma^C$ . The line shape of  $Z^+$  in the  $D^*\bar{D}^*\pi$  channels can be obtained by inserting Eq. (8) into Eq. (10) and using the expression for  $\text{Im}\kappa(E)$  in Eq. (7):

$$\frac{d\Gamma}{dE} \left[ B^+ \to K^0 + D^* \bar{D}^* \pi \right] 
= 2\Gamma_B^K |f(E)|^2 \sqrt{M_{1*}} \left[ (E^2 + \Gamma_1^2/4)^{1/2} + E \right]^{1/2}, (12)$$

where  $D^*\bar{D}^*\pi$  denotes  $D^{*+}D^{*-}\pi^+$  or  $D^{*+}\bar{D}^{*0}\pi^0$  or  $D^{*0}\bar{D}^{*0}\pi^+$ . Since  $\bar{D}^0_1$  decays primarily into  $D^{*-}\pi^+$  and  $\bar{D}^{*0}\pi^0$  with branching fractions  $\frac{2}{3}$  and  $\frac{1}{3}$  and  $D^+_1$  decays primarily into  $D^{*+}\pi^0$  and  $D^{*0}\pi^+$  with branching fractions  $\frac{1}{3}$  and  $\frac{2}{3}$  [20], the line shapes in any of the three individual  $D^*\bar{D}^*\pi$  channels can be obtained by multiplying Eq. (12) by 1/3. The line shapes given in Eqs. (11) and (12) require only that the Z has  $J^P=0^-$ ,  $1^-$ , or  $2^-$ . The spin J determines the correlations between the polarizations of the two spin-1 constituents of  $Z^+$ .

We proceed to illustrate the line shapes of the Z. We

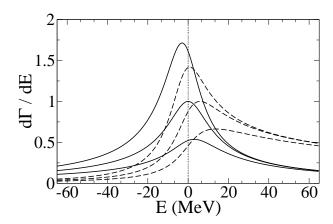


FIG. 1: Line shapes of the  $Z^+$  in the  $\psi'\pi^+$  decay channel (solid lines) and in  $D^*\bar{D}^*\pi$  decay channels (dashed lines) for three values of the binding energy:  $E_Z=+3$  MeV (upper lines), 0 MeV (middle lines), and -3 MeV (lower lines). The line shapes are normalized so that their maximum values are 1 for  $E_Z=0$ .

will assume that decays of Z into  $D^*\bar{D}^*\pi$  dominate over the short-distance decays. We can therefore neglect Im $\gamma$  in the resonance factor  $|f(E)|^2$  in the line shapes for  $\psi'\pi$  and  $D^*\bar{D}^*\pi$  in Eqs. (11) and (12). Thus the only unknown parameters are Re $\gamma$  and the normalization factors  $\Gamma_B^K$  and  $\Gamma^{\psi'\pi}$ . We identify the binding energy  $E_Z$  with the negative of the position of the peak in the line shape for  $\psi'\pi$ . It satisfies the equation

$$E_Z = \frac{\text{Re}\gamma}{2\sqrt{M_{1*}}} \left[ (E_Z^2 + \Gamma_1^2/4)^{1/2} + E_Z \right]^{1/2}.$$
 (13)

The line shapes of  $Z^+$  in  $\psi'\pi^+$  and  $D^*\bar{D}^*\pi$  are shown in Fig. 1 for three values of the binding energy:  $E_Z = +3$ , 0 and -3 MeV, which correspond to Re $\gamma = +53.9$ , 0, and -72.0 MeV, respectively. The normalization factors  $\Gamma_B^K$ and  $\Gamma^{\psi'\pi}$  were chosen so that the two line shapes have the same peak value for  $E_Z = 0$ . These same factors were then used for  $E_Z = \pm 3$  MeV. The peaks in the line shape are most dramatic if  $E_Z$  is positive. The line shapes in  $\psi'\pi$  are nearly symmetric about the peak at the energy  $-E_Z$ . If  $E_Z < 0$ , the line shape is sometimes referred to as a "cusp" [4] because it has a discontinuity in slope at E=0 if  $\Gamma_1=0$ . In the case of  $Z^+$ , the cusp is completely smoothed out by the width of the  $D_1$ , as is evident in the lowest solid line in Fig. 1. The line shapes in  $D^*D^*\pi$  have a peak at a higher energy than for  $\psi'\pi$ and they also have a broad shoulder on the high energy side of the peak. This behavior can be attributed to a threshold enhancement in the production of  $D_1\bar{D}^*$  and  $D^*\bar{D}_1$  that overlaps with the resonance.

To quantify the behavior of the line shapes in Eqs. (11) and (12), we can exploit the fact that the binding energy  $E_Z$  is small compared to the width  $\Gamma_1$  of the constituent. Eq. (13) for  $E_Z$  can be solved as an expansion in powers of Re $\gamma$  whose leading term is  $[\Gamma_1/(8M_{1*})]^{1/2}$ Re $\gamma$ . The expansion can be inverted to give Re $\gamma$  as an expansion

in  $E_Z/\Gamma_1$ . To first order in  $E_Z/\Gamma_1$ , the energy at which the line shape for  $D^*\bar{D}^*\pi$  has its maximum is

$$E_{\text{max}}^{D^* \bar{D}^* \pi} \approx 0.289 \,\Gamma_1 - 2.03 \,E_Z \,.$$
 (14)

Since  $|E_Z| \ll \Gamma_1$ , the difference between the locations of the peaks in the  $D^*\bar{D}^*\pi$  and  $\psi'\pi$  line shapes is approximately  $\Gamma_1/\sqrt{12} \approx 6\,$  MeV. The line shapes in Eqs. (11) and (12) are not Breit-Wigner resonances, so they can not be characterized simply by a width  $\Gamma_Z$ . A simple way to quantify their widths is to give the energies at which the line shapes decrease to half of their maxima. For the  $\psi'\pi$  line shape, the half-maxima are at the energies

$$E_{+}^{\psi'\pi} \approx 0.866 \,\Gamma_1 - 3.42 \,E_Z \,,$$
 (15a)

$$E^{\psi'\pi} \approx -0.866 \,\Gamma_1 + 0.16 \,E_Z \,.$$
 (15b)

The full width at half maximum is approximately  $\sqrt{3} \Gamma_1 \approx 35$  MeV, which is consistent with the measured width of  $Z^+$  in Eq. (1b). For the  $D^*\bar{D}^*\pi$  line shape, the half-maxima are at the energies

$$E_{+}^{D^*\bar{D}^*\pi} \approx 3.017 \,\Gamma_1 - 15.37 \,E_Z \,,$$
 (16a)

$$E_{-}^{D^*\bar{D}^*\pi} \approx -0.371 \Gamma_1 - 1.08 E_Z$$
 (16b)

The full width at half-maximum is approximately  $3.4 \Gamma_1 \approx 69$  MeV, which is about twice the width in  $\psi'\pi$ . The right half-maximum is about 4 times as far from the peak energy as the left half-maximum.

The universal scattering amplitude in Eq. (5) also applies to the X(3872) if it is a  $D^*\bar{D}$  molecule [16]. The difference between the lifetimes of the constituents of the

X and  $Z^{\pm}$  leads to a significant difference in their line shapes. In the case of the X, the  $D^{*0}$  has a width of about 65 keV, which is small compared to the 8 MeV splitting between the  $D^{*0}\bar{D}^0$  and  $D^{*+}D^-$  thresholds. Since the binding energy of X relative to the  $D^{*0}\bar{D}^0$ threshold is small compared to 8 MeV, it is a  $D^{*0}\bar{D}^{0}$ molecule with a very small  $D^{*+}D^{-}$  component. The line shapes of the X are given by simple universal formulas analogous to those in Eqs. (11) and (12) only if the energy is within a few MeV of the  $D^{*0}\bar{D}^{0}$  threshold. For energies more than a few MeV from the threshold, one must take into account the resonant coupling to the charged charm meson channel  $D^{*+}D^{-}$  [21, 22]. The width of the  $D^*$  provides less smearing than that of the  $D_1$ , so it is possible for the line shape for the X to have a two-peaked structure consisting of a resonance and a threshold enhancement.

In summary, the hypothesis that the  $Z^{\pm}$  is a weakly-bound S-wave charm meson molecule implies that its line shape in  $D^*\bar{D}^*\pi$  should peak at a higher energy and have a larger width than its line shape in  $\psi'\pi$ . If our quantitative predictions for these line shapes are confirmed, it would provide strong support for this interpretation of Z. The tiny binding energy of the Z requires an accidental fine-tuning of the charm quark mass. The existence of the Z suggests that there may be  $c\bar{c}$  tetraquark mesons in other  $J^P$  and  $I^G$  channels that are more strongly bound. It also implies that there must be a  $b\bar{b}$  analog of the Z with a much larger binding energy [23, 24]. Thus the discovery of the Z(4430) is just the beginning of the spectroscopy of tetraquark mesons.

This research was supported in part by the Department of Energy under grant DE-FG02-91-ER40690.

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